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Kelvin-Helmholtz discontinuity in partially ionized plasmas

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Abstract : This paper treats the linear Kelvin-Helmholtz discontinuity in partially ionized plasmas in a uniform vertical magnetic field. The dispersion relation has been derived by using the normal mode technique. Both the streaming velocity and the frequency collision are found to have a destabilizing influence on the instability of system.

Keywords : Plasma instability, Kelvin-Helmholtz discontinuity, dispersion relation.

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1. Introduction

The problem of instability of a plane interface between two superposed fluids which are in relative horizontal motion parallel to their interface is known as the Kelvin-Helmholtz instability. The K-H instability occurs in situations such as when air is blown over mercury or when highly ionized hot plasma is surrounded by a slightly cold ionized gas. From geophysical point of view, these situations arise when a meteor enters the Earth's atmosphere. Investigations for such a system have been analyzed by several researchers in the past *e.g.* Chang and Russel [1], D'Angelo and Von Goeler [2], Willison and Chang [3], Singh and Tandon [4], Kalra, *et al* [5]. The K-H instability of superposed plasmas has been considered by Shivamoggi [6]. He has shown that in the case of plasma immersed in a horizontal magnetic field, the inclusion of finite resistivity leads to new unstable modes. However, in the later stages of the derivation of the dispersion relation, he has assumed the densities of two plasmas to be equal. The K-H instability problem of superposed fluids has attracted the attention of several researchers in the recent years under different assumptions. Malik and Singh [7] have stuided the problem of chaos in K-H instability in magnetic fluids by using the bifurcation method and applying Melnikov function. The importance of the Kelvin-Helmholtz problem has been demonstrated by Benjamin and Bridges [8,9]. They have given an excellent reappraisal of the classic Kelvin-Helmholtz problem in hydrodynamics and have shown that the problem admits of a canonical Hamiltonian formulation and obtained several new results. Allah [10] has investigated the effects of magnetic field and heat and mass transfer on the Kelvin-Helmholtz instability of superposed fluids. More recently, Bhatia and Sharma [11] have studied the effects of surface tension and permeability of the porous medium on Kelvin-Helmholtz instability of superposed viscous conducting fluids in a uniform vertical magnetic field.

In cosmic physics there are several situations such as in chromosphere, solar photosphere and in cool interstellar clouds where the plasmas are not frequently fully ionized but may instead be partially ionized so that interaction between the ionized fluid and neutral gas becomes important. The study of instability of partially ionized plasma has been carried out by several researchers in the past *e.g.* Gupta and Bhatia [12] have studied Rayleigh-Taylor instability of superposed viscous partially ionized plasmas in a horizontal magnetic field. It would therefore,

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be of interest to examine the Kelvin-Helmholtz discontinuity in partially ionized superposed plasmas in the presence of effects of ion viscosity in vertical magnetic field. This aspect forms the basis of this paper.

2. Linearized perturbation equations

We consider the motion of the mixture of an incompressible, infinitely conducting viscous fluid of density ρ and a neutral gas of density $\rho_d (\rho_d << \rho)$ having a streaming velocity V along the horizontal direction.

(i) The magnetic field is taken to be uniform and acting along a vertical direction. We assume that the magnetic field interacts with the conducting fluid,

(ii) the steady state velocities of the two components (ionized fluid and neutral gas) are equal,

(iii) the effects of the pressure gradient and gravity are ignored on the neutrals and

(iv) both the components behave like continuums and flow with the same velocity. The model considered for partially ionized plasma is highly idealized and is the same as considered in earlier studies *e.g.* Bhatia [13] and it is hoped that it will nevertheless reveal the essential features of the considered effect on the plasma instability problem.

Retaining only the linear terms in the perturbed quantities in the governing equations and making use of the above assumptions, we get the linearized perturbation equations :

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho (\boldsymbol{V} . \nabla) \boldsymbol{u} = -\nabla \delta p + (\nabla \times \boldsymbol{h}) \times \boldsymbol{H} + g \delta \rho +$$

$$\mu \nabla^2 \boldsymbol{u} + \rho_d \upsilon_c (\boldsymbol{u}_d - \boldsymbol{u}), \quad (1)$$

$$\frac{\partial}{\partial t} \boldsymbol{u}_d + (\boldsymbol{V} . \nabla) \boldsymbol{u}_d = -\upsilon_c (\boldsymbol{u}_d - \boldsymbol{u}), \quad (2)$$

$$\frac{\partial}{\partial t} (\delta \rho) + (\boldsymbol{V} . \nabla) \delta \rho + (\boldsymbol{u} . \nabla) \rho = 0, \quad (3)$$

$$\frac{\partial \boldsymbol{h}}{\partial t} + (\boldsymbol{V} . \nabla) \boldsymbol{h} = (\boldsymbol{H} . \nabla) \boldsymbol{h}, \quad (4)$$

$$\nabla \boldsymbol{h} = 0, \quad (5)$$

$$\nabla \boldsymbol{u} = 0, \quad (6)$$

where $\boldsymbol{h} = (h_x, h_y, h_z)$, $\delta \rho$ and δp are respectively the perturbations in the magnetic field $\boldsymbol{H}$, density ρ and pressure p of the ionized fluid, $\boldsymbol{u}$ = (u, v, w) is the velocity of the ionized fluid while $\boldsymbol{u}_d$ denotes the velocity of neutral gas. Here μ is the coefficient of viscosity and υ_c is the collision frequency between the two components frequency between the two components of the plasma.

We take streaming velocity $\boldsymbol{V}$ = $(U,0,0)$ and the magnetic field $\boldsymbol{H}$ = $(0,0,H)$. For longitudinal wave propagations, we analyse the disturbance in terms of normal modes and seek solutions depending on space coordinates x, z and time t of the form

$$F(z) \ \exp(ik_x x + nt), \quad (7)$$

where $F(z)$ is the some function of z and k_x is the wave number along the x-direction and n, may be complex, denotes the rate which system disturbs from the equilibrium.

Eliminating some of the variables from the above equations and on writing $\frac{d}{dz} = D$, $\beta = \frac{\rho}{\rho_d}$, we finally obtain an equation in w

$$\frac{H^2 (D^2 - k^2) D^2 w}{n'} - \rho n_c (D^2 - k_x^2) w - \frac{g (D\rho) k_x^2 w}{n'}$$

$$\cdot \mu (D^2 - k_x^2) w = 0 ,$$

where

$$n_c = n' \left| 1 + \frac{\beta \nu_c}{n' + \nu_c} \right| \ \text{and} \ n' = n + ik_x U . \quad (8)$$

3. Two superposed streaming plasmas

We now consider the case when two superposed plasmas of uniform densities ρ_1 and ρ_2, uniform viscosities μ_1 and μ_2, magnetic field H_1 and H_2 and with streaming velocities U_1 and U_2, are separated by a horizontal boundary z = 0. Then in both the regions of constant density, eq. (8) becomes

$$(D^2 - k_x^2)(D^2 - M^2) w = 0, \quad (9)$$

where

$$M^2 = \left(k_x^2 + \frac{n_c}{\nu} \right) \left(1 + \frac{V^2}{n' \nu} \right)^{-1} \quad (10)$$

Here, $\nu = \frac{\mu}{\rho}$ and $V^2 = \frac{H^2}{\rho}$ are the kinematic viscosity and Alfvén velocity respectively.

Now seeking the solution of the eq. (9) which remains bounded in the two regions, we obtain

$$w_1 = A_1 n_{c1} e^{k_x z} + B_1 n_{c1} e^{m_1 z}, \quad z < 0; \tag{11}$$

$$w_2 = A_2 n_{c2} e^{-k_x z} + B_2 n_{c2} e^{-m_2 z}, z < 0; \tag{12}$$

where A_1, B_1, A_2 and B_2 are constants and m_1 and m_2 are positive square roots of eq. (10) for the two regions and

$$_{c1} = n_1' \left| 1 + \frac{\beta v_c}{n_1' + v_c} \right. \tag{13}$$

and a similar expression for n_{c2}.

The above solutions must satisfy certain boundary conditions. These boundary conditions require that at the interface $z = 0$. If we integrate eq. (8) across the interface $z = 0$, we obtain another condition. If we integrate eq. (8) across the interface $z = 0$, we obtain another conditions.

$$w, Dw \text{ and } \mu\left(D^2 + k_x^2\right)w \tag{14}$$

must be continuous. These conditions follow from the requirements that the horizontal and vertical components of velocity and the tangential viscous stresses must be continuous across the interface.

$$\left[n_{c2}\rho_2 Dw_2 - \mu_2\left(D^2 - k_x^2\right)Dw_2\right] + \frac{H^2}{n_2'}\left(D^2 - k_x^2\right)Dw_2 \Big|_{z=0}$$
$$\left[n_{c1}\rho_1 Dw_1 - \mu_1\left(D^2 - k_x^2\right)Dw_1\right] + \frac{H^2}{n_1'}\left(D^2 - k_x^2\right)Dw_1 \Big|_{z=0}$$
$$= -gk_x^2\left(\frac{\rho_2}{n_2'} - \frac{\rho_1}{n_1'}\right)w_0 - 2k_x^2(\mu_2 - \mu_1)Dw_0 \tag{15}$$

where w_0 and Dw_0 are unique values of their quantities at $z = 0$ and if we apply the conditions (14) and (15) to the solutions (11) and (12), we get

$$A_1 + B_1 = A_2 + B_2, \tag{16}$$

$$K_x A_1 + M_1 B_1 = -K_x A_2 - M_2 B_2, \tag{17}$$

$$\mu_1[2k_x^2 A_1 + (M_1^2 + k_x^2)B_1] = \mu_2[2k_x^2 A_2 + (M_2^2 + k_x^2)B_2] \tag{18}$$

$$-k_x\rho_2 n_{c2} - k_x\rho_1 n_{c1} = \frac{gk_x^2}{2}\left(\frac{\rho_2}{n_2'} - \frac{\rho_1}{n_1'}\right)(A_1 + B_1 + A_2 + B_2)$$
$$-k_x^2(\mu_2 - \mu_1)(k_x A_1 + M_1 B_1 - k_x A_2 - M_2 B_2) \tag{19}$$

Eliminating the constants from eqs. (16) to (19), we obtain a characteristic equation

$$(M_1 - k_x)\left[2k_x^2(\alpha_1 \upsilon_1 - \alpha_2 \upsilon_2)\left\{n_{c2}\alpha_2 + \frac{C}{k_x}(M_2 - k_x)\right\} + \right.$$

$$\alpha_2\upsilon_2\left(M_2^2 - k_x^2\right)(R - n_{c1}\alpha_1 - n_{c2}\alpha_2)\Big] - 2k_x\Big[\alpha_1\upsilon_1\left(M_1^2 - k_x^2\right)$$

$$n_{c2}\alpha_2 + \frac{C}{k_x}(M_2 - k_x)\Big\},$$

$$+\alpha_2\upsilon_2\left(M_2^2 - k_x^2\right)\left[\left[\left\{n_{c1}\alpha_1 - \frac{C}{k_x}(M_1 - k_x)\right\}\right]\right]$$

$$(M_2 - k_x)\Big|\alpha_1\upsilon_1\left(M_1^2 - k_x^2\right) \times (R - n_{c1}\alpha_1 - n_{c2}\alpha_2)$$

$$\left. - 2k_x^2(\alpha_1\upsilon_1 - \alpha_2\upsilon_2)\left\{n_{c1}\alpha_1 - \frac{C}{k_x}(M_1 - k_x)\right\}\right]\right] = 0, \tag{20}$$

where $\alpha_{1,2} = \dfrac{\rho_{1,2}}{\rho_1 + \rho_2}$, $R = gk \left(\dfrac{\alpha_1}{n} \quad \alpha_2 \right.$

and $C = k_x^2(\alpha_1\upsilon_1 - \alpha_2\upsilon_2)$. (21)

The dispersion relation (20) is quite complex particularly as M_1 and M_2 involve square roots. We therefore, carry out the study for the case of highly viscous superposed plasmas for which we can write

$$M_{1,2} = k_x + \frac{n_{c1,2}}{2k_x\upsilon_{1,2}} \quad \frac{k_x V_{1,2}}{2n_{1,2}'\upsilon_{1,2}} \tag{22}$$

on neglecting second and higher order terms in $1/\upsilon_{1,2}$ *i.e.* assuming both the plasmas to be highly viscous. Substituting the values of M_1 and M_2 from (22) in eq. (20) and simplifying we get the dispersion relation

$$\sum_{i=0}^{11} A_i n^i = 0, \tag{23}$$

where the coefficients A_i's are complex and involve $U_{1,2}$, $\upsilon_{1,2}$, $\upsilon_{c1,2}$, $V_{1,2}$ and $\alpha_{1,2}$. The coefficients are not given here as they are very lengthy expressions.

4. Discussion

It is not easy to obtain analytical results from eq. (23) as it is quite complex. We have therefore, solved it numerically. Eq. (23) simplifies considerably when we consider the case of two streams of equal kinematic viscosities and same collision frequencies, flowing past each other in opposite directions with equal velocities *i.e.*

$$U_1 = U,\ U_2 = -U,\ \upsilon_1 = \upsilon_2 = \upsilon,\ \upsilon_{c1} = \upsilon_{c2} = \upsilon_c. \quad (24)$$

On writing

$$n^* = n/\sqrt{g}\ ,\ \upsilon^* = \upsilon/\sqrt{g}\ ,\ \upsilon_2^* = \upsilon/\sqrt{g}\ ,\ U = U/\sqrt{g},$$

$$H_{1,2}^{*2} = H_{1,2}^2/\sqrt{g}\ ,\ V_{1,2}^{*2} = V_{1,2}^2/\sqrt{g} \quad (25)$$

in eq. (23), the dispersion relation is non-dimensionalized. The choice of the values of H_1^*, H_2^* and α_1, α_2 can be made freely and then the values of V_1^*, V_2^* can be calculated.

For numerical estimates of the roots n^* from non-dimentionalized eq. (23), we set $\alpha_1 = 0.25$, $\alpha_2 = 0.75$ (unstable configuration), $H^{*2}{}_1 = 0.25$, $H^{*2}{}_2 = 3$. Then $V^*{}_1 = 1$, $V^*{}_2 = 2$. The non-dimensionalized eq. (23) has been solved numerically for serveral values of the parameters U^* and υ_c^* for fixed values of the other parameters to examine the dependence on n^* of U^* and υ_c^*. These calculations are presented in Figure 1 where we plot the growth rate n^* of the unstable mode against wave number k_x for U^* (streaming velocity) = 5, 10, 15 taking fixed υ_c^*. The figure also presents the growth rate for $\upsilon_c^* = 0.1$, 0.2, 0.3 when U^* is kept fixed. In all these calculations,

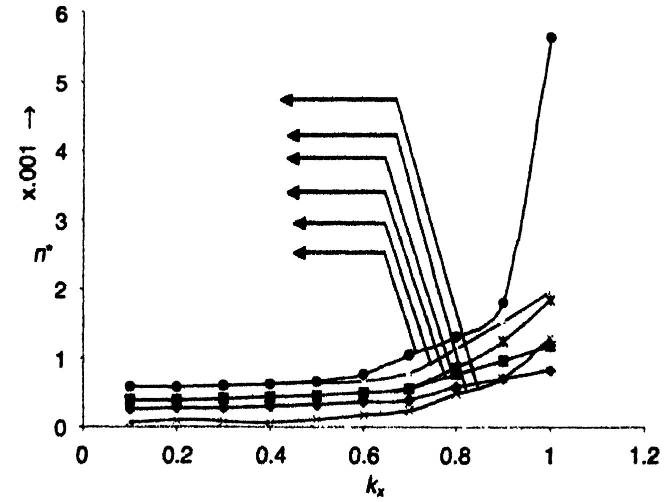

Figure 1. Plot of growth rate n^* against the wave number k_x for $v_c^* = 0.1, 0.2, 0.3$ and $U^* = 5, 10, 15$ taking $\alpha_1 = 0.25$, $\alpha_2 = 0.75$ and $V_1^* = 1.0$, $V_2^* = 2.0$.

$\beta = \rho_d/\rho$ (the ratio of the densities of the neutral gas and the ionized fluid) = 0.1. It is seen from the calculations presented here that the growth rate n^* increases when both U^* and υ_c^* increase for constant k_x. The effects of streaming motion and collision frequency are therefore both destabilizing. It is also seen that the growth rate n^* is higher for large wave numbers and is smaller for small wave numbers ($n = 0$ when $k_x = 0$). Further, the increase in n^* with the increase in the values of U^* and υ_c^* is smaller for small wave numbers and is larger for large wave numbers. The unstable mode of disturbance with small wave length is thus much more destablized by U^* and υ_c^* than the disturbance with large wave length.

Bhatia and Steiner [14] have shown that effects of streaming motion and collision frequency are both destabilizing on the Kelvin–Helmholtz discontinuity of superposed inviscid plasmas in a horizontal magnetic field. The results obtained in this paper are therefore, in agreement with earlier observations of the Kelvin-Helmholtz instability in superposed fluids.

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